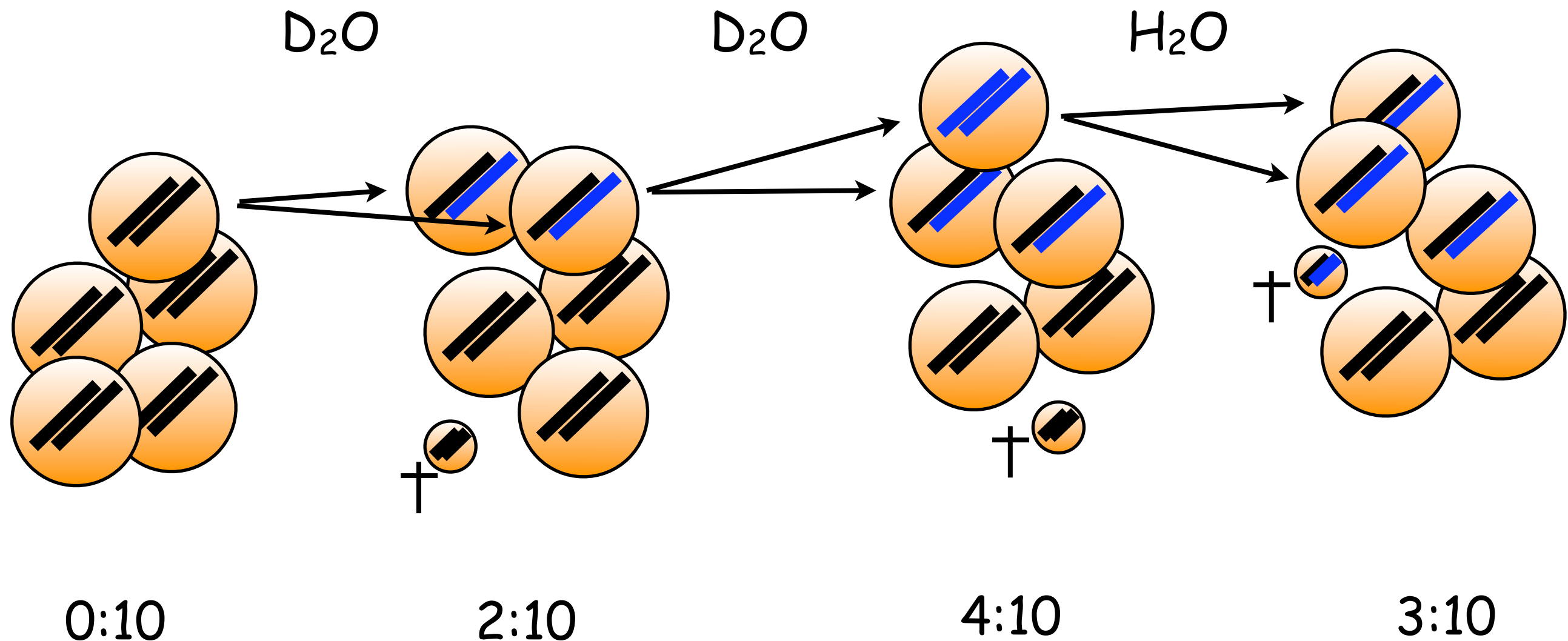


Systems Immunology
(Quantitative Immunology)
requires estimating many parameters.
This can be challenging.

Rob de Boer
Theoretical Biology, Utrecht University, NL

Quantification example 1: Deuterium labeling

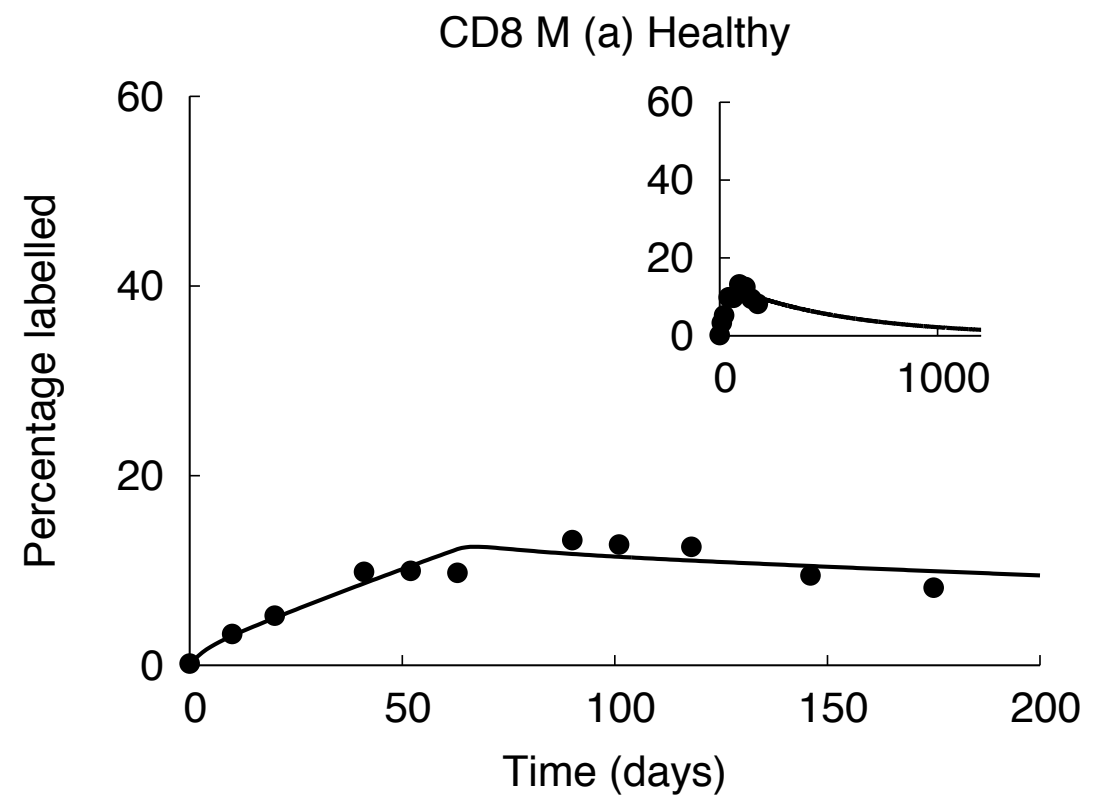
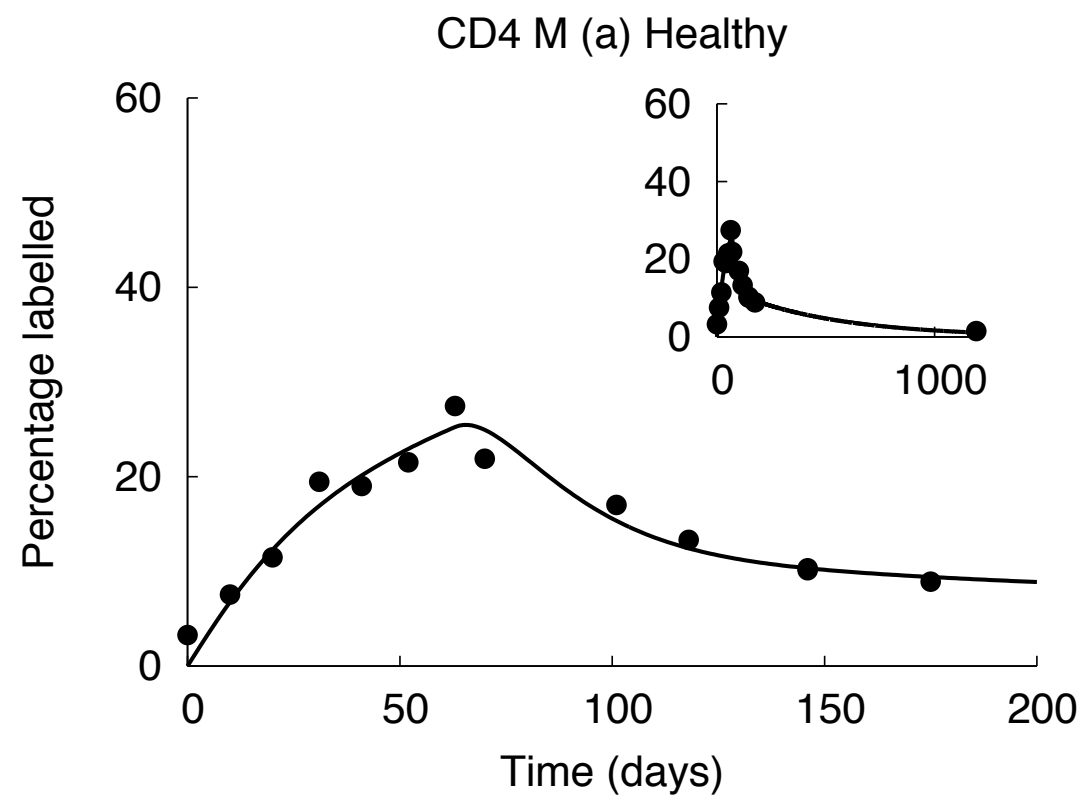
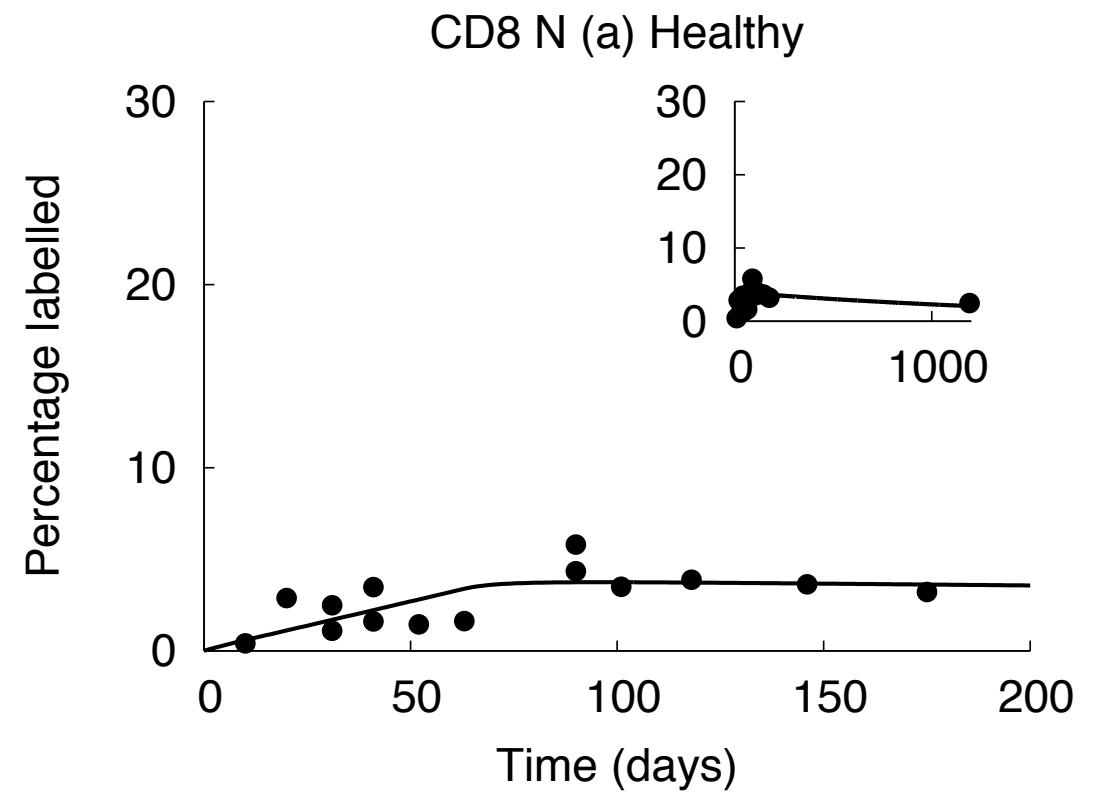
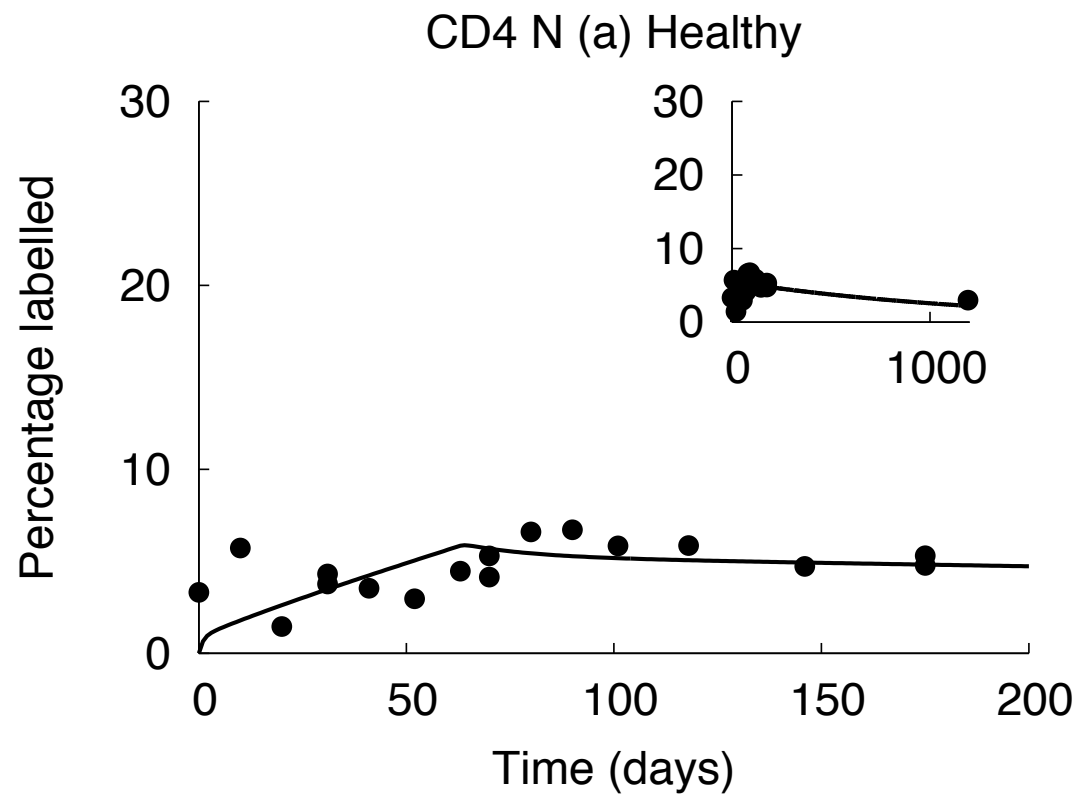


In the presence of deuterium, cell division copies DNA strands into labeled DNA strands: $U \rightarrow U + L$ and $L \rightarrow L + L$

In its absence $U \rightarrow U + U$ and $L \rightarrow L + U$

DNA strands can only disappear by cell death

Healthy human volunteers: one individual (a)



Results from 5 human volunteers

Expected life spans

Naive CD4⁺ T cells: 2300 days (6.2 years)

Naive CD8⁺ T cells: 3300 days (9.1 years)

Memory CD4⁺ T cells: 160 days (0.45 years)

Memory CD8⁺ T cells: 120 days (0.33 years)

Compartments:

Fitting the naive T cell data typically requires only one compartment: no evidence for short-lived RTE

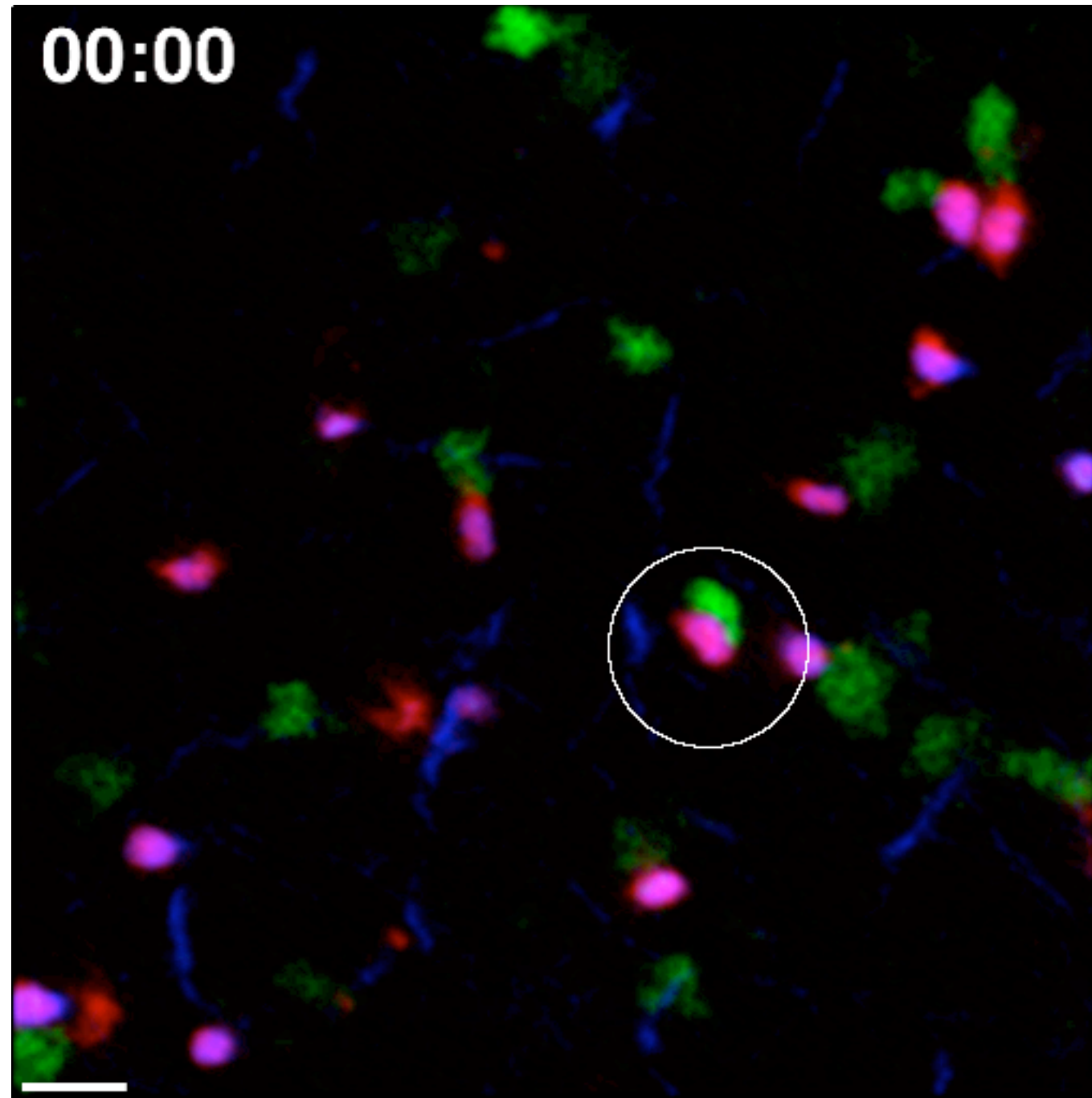
Memory data do require 2 compartments: heterogeneity

Similar results for mice but 50-fold faster

Borghans, Vrisekoop, Den Braber, Mugwagwa, Tesselaar, Miedema

Quantification 2: killing rates of CTL:

2PM movie of Ag pulsed B cells being killed by CTL

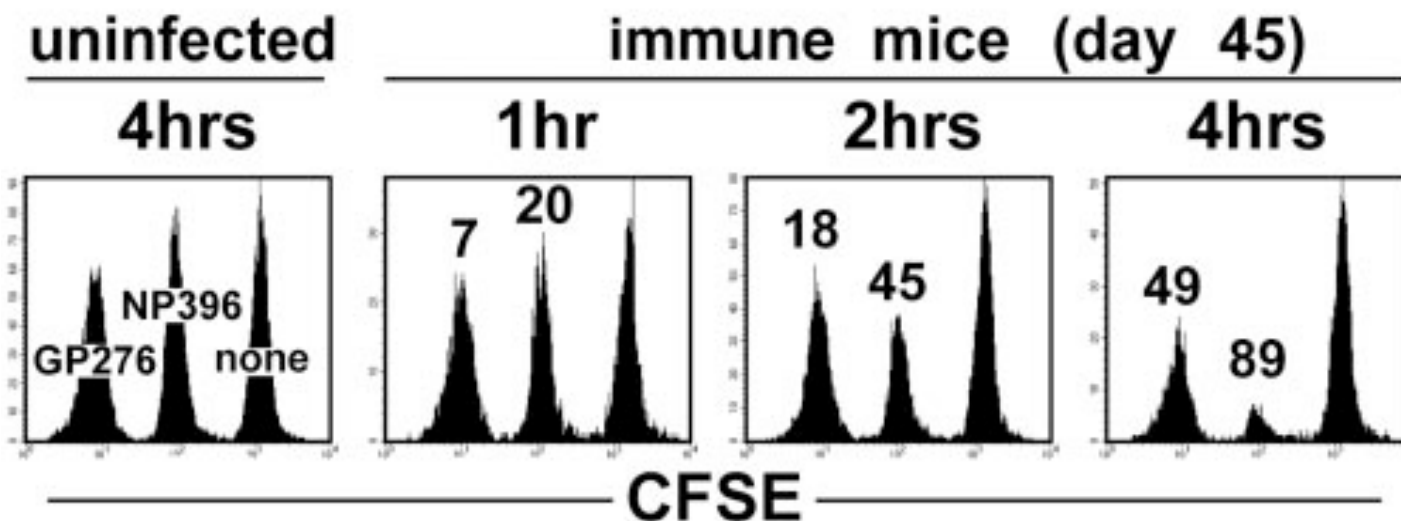
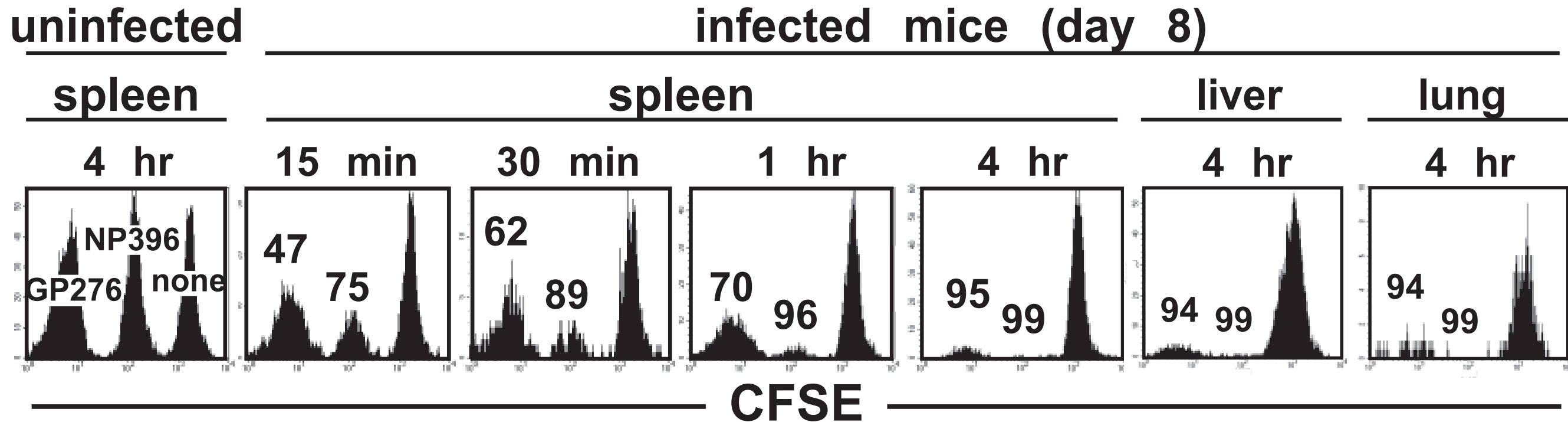


B cell (target cell): purple, CTL: green, death B cell: white

From: Mempel et al. Immunity 2006

Adoptive transfer experiments: Barber et al JI03

- transfer peptide pulsed splenocytes into mice (GP276 & NP396)
- at peak of LCMV response (d8) or in memory phase (d45)
- compare numbers of pulsed and unpulsed cells in spleen



Numbers give percentage
target cells killed
high E:T ratio

Very rapid killing of target cells

Modeling the Barber et al data

Antia, Regoes,
Yates, Barber,
Graw, Ganusov,
De Boer

$$T' = \sigma B - (e + K)T$$

Death rates K :

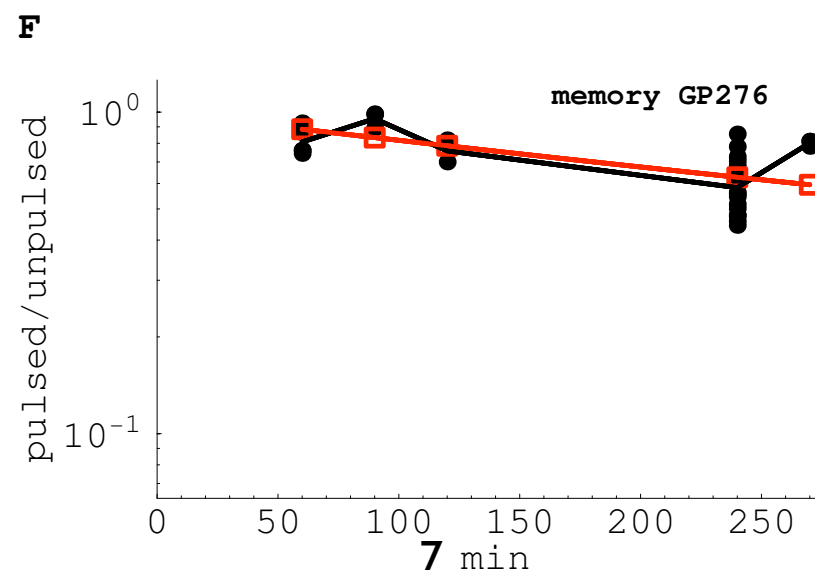
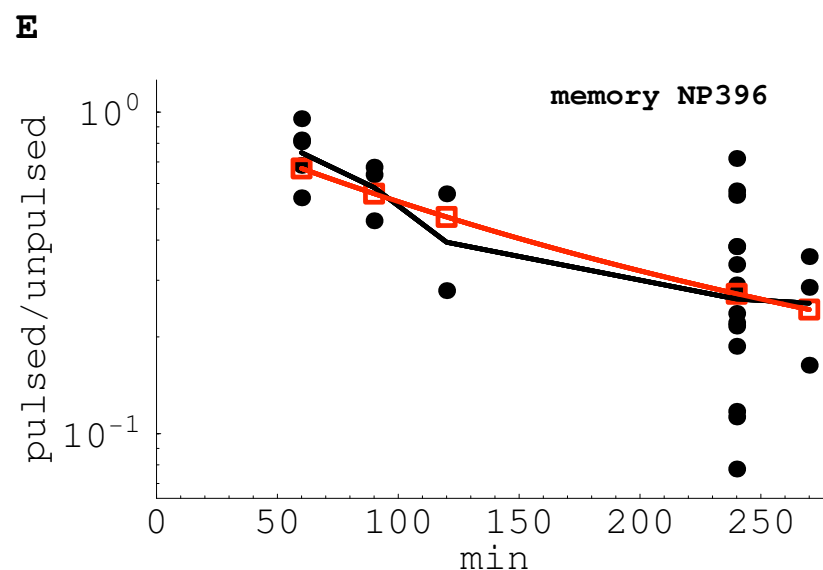
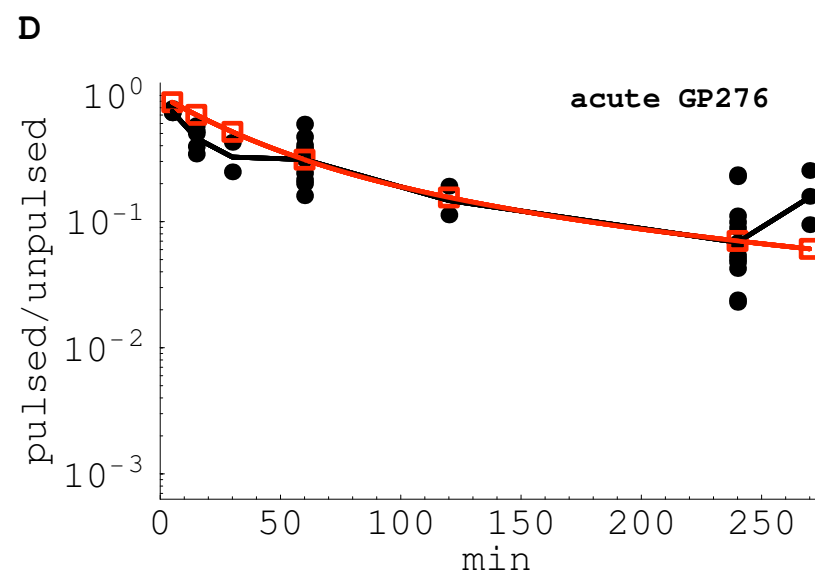
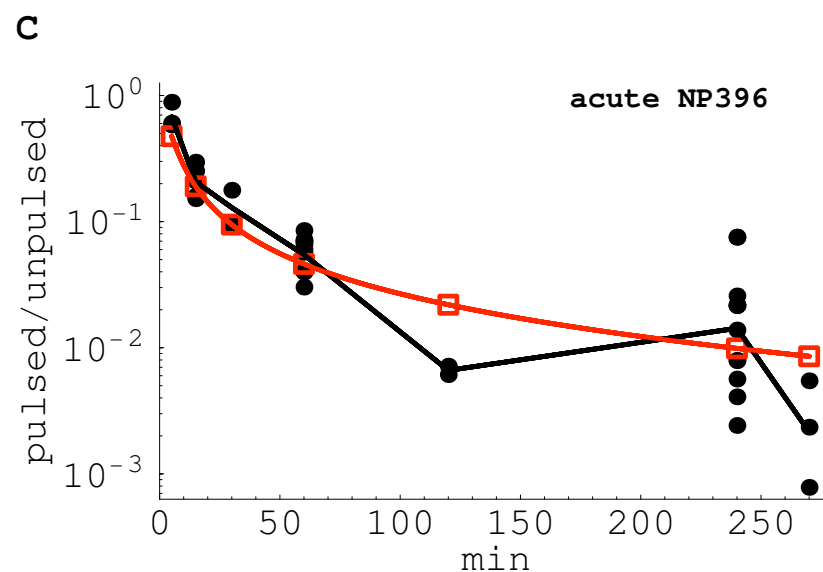
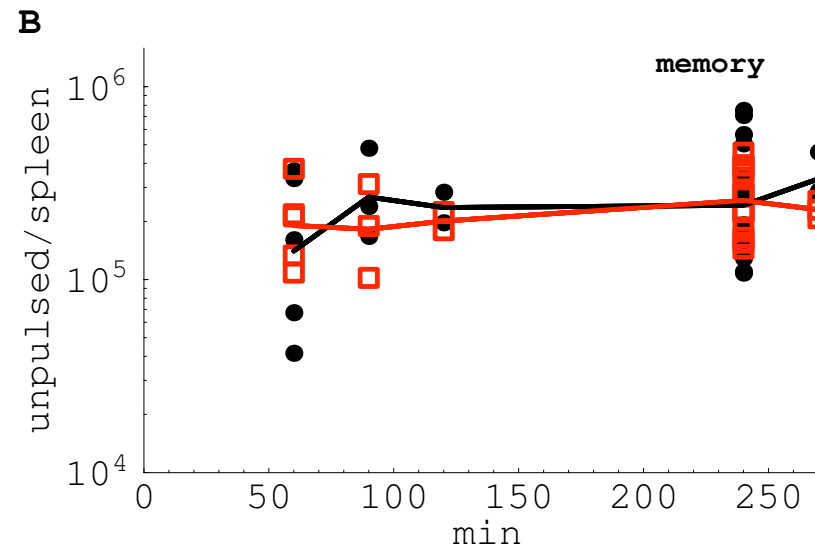
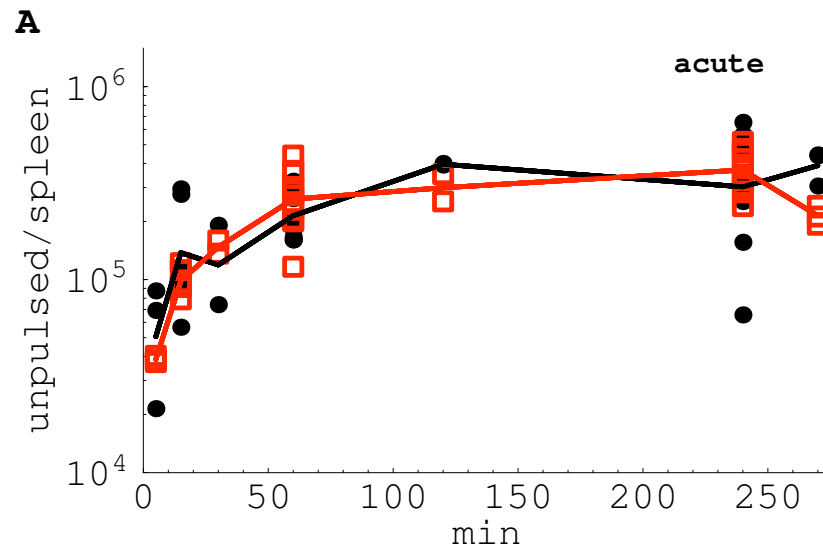
$$K_{NP}^a = 497 \text{ d}^{-1}$$

$$K_{GP}^a = 72 \text{ d}^{-1}$$

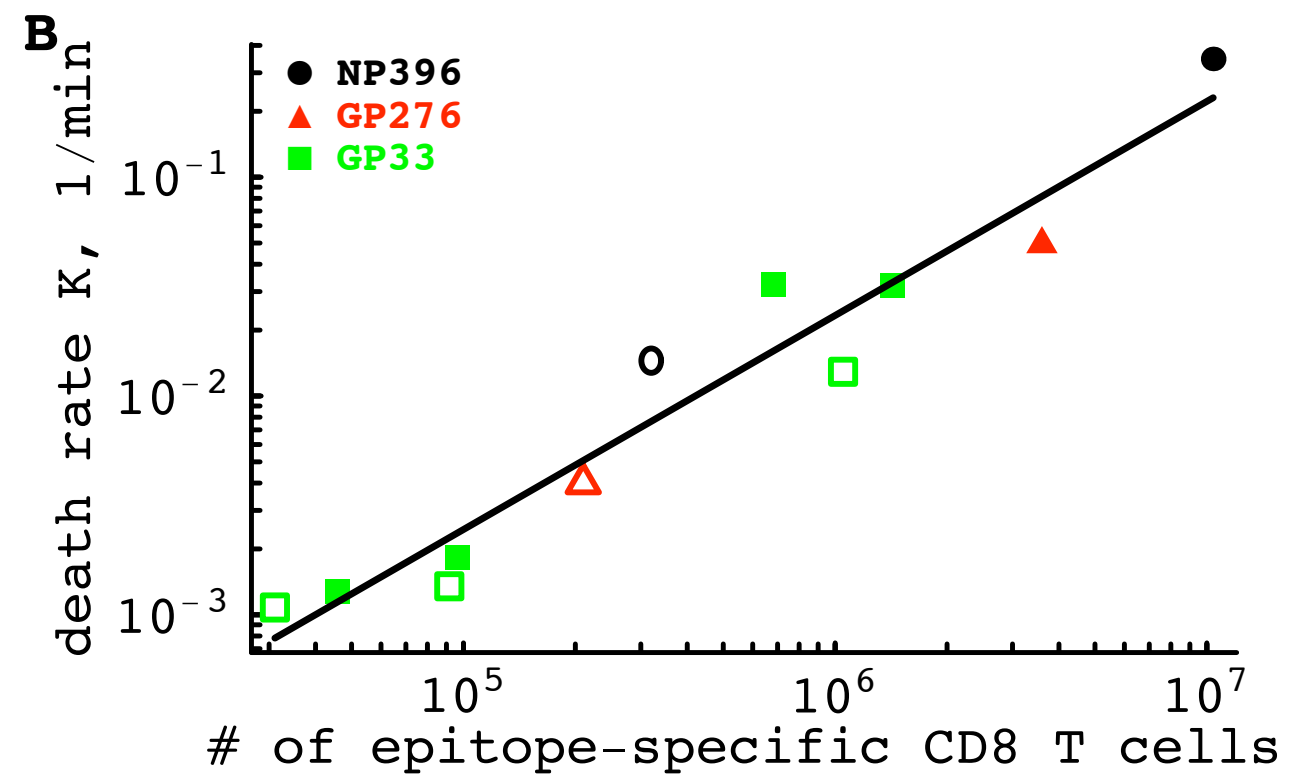
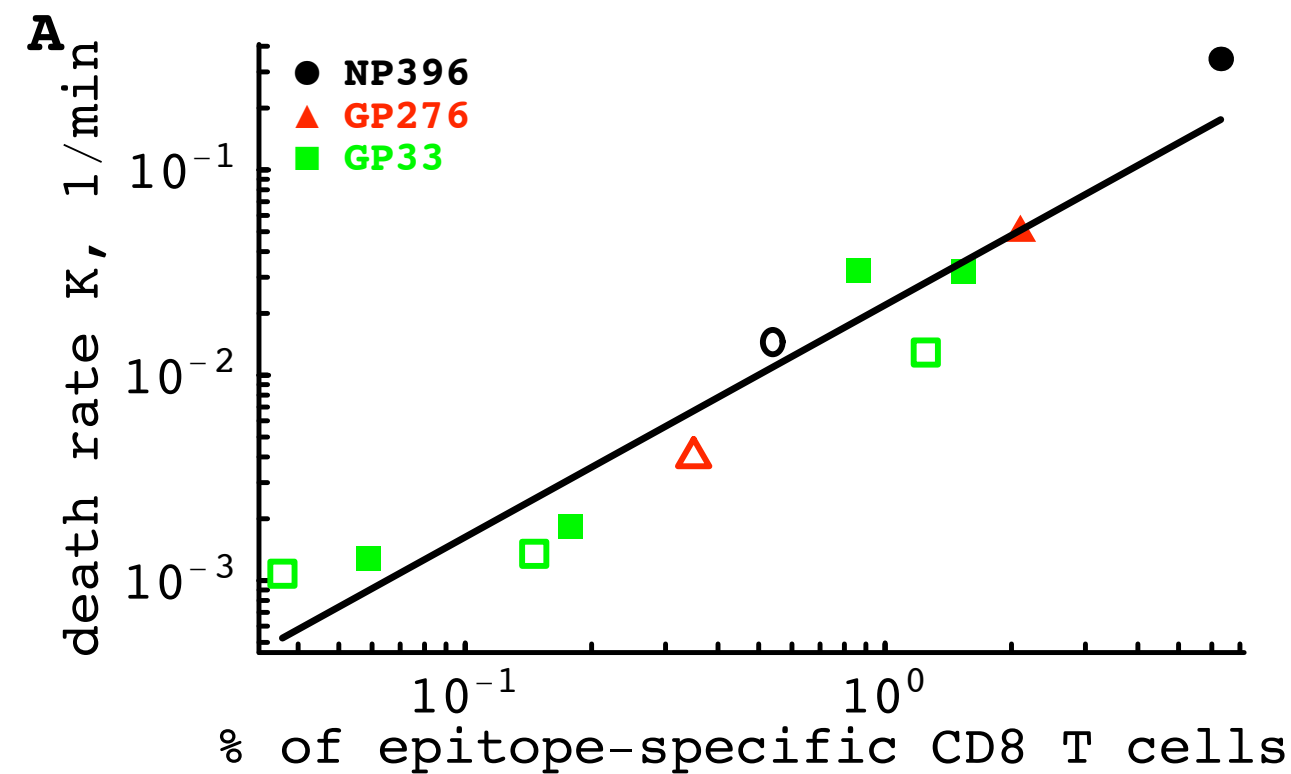
$$K_{NP}^m = 21 \text{ d}^{-1}$$

$$K_{GP}^m = 6 \text{ d}^{-1}$$

500 d^{-1} is 3 min



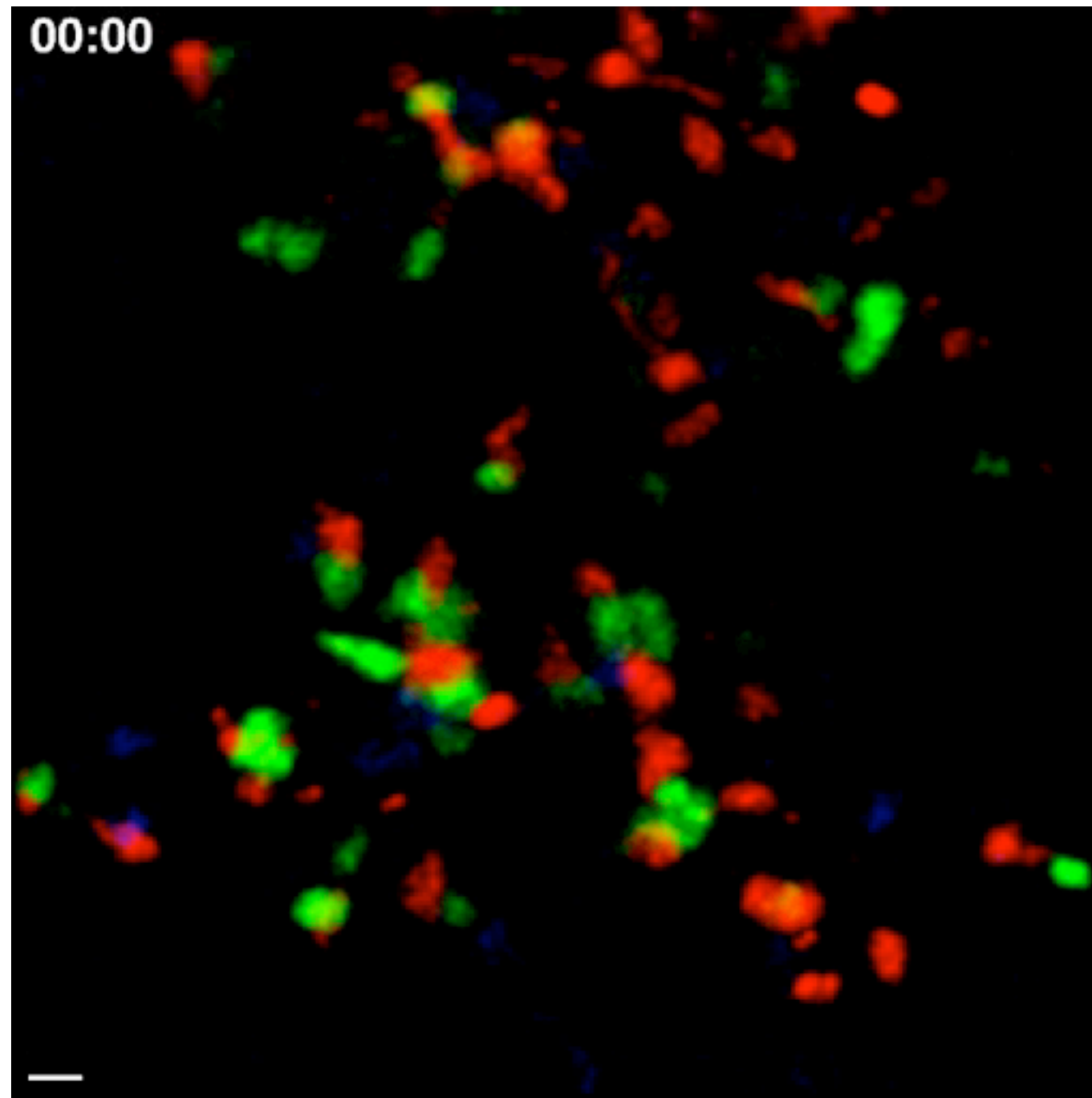
Modeling more Barber et al data



Killing seems to follow a mass action term
differences between epitopes seems small
One CTL kills $KT/E=1-5$ target cells per day.

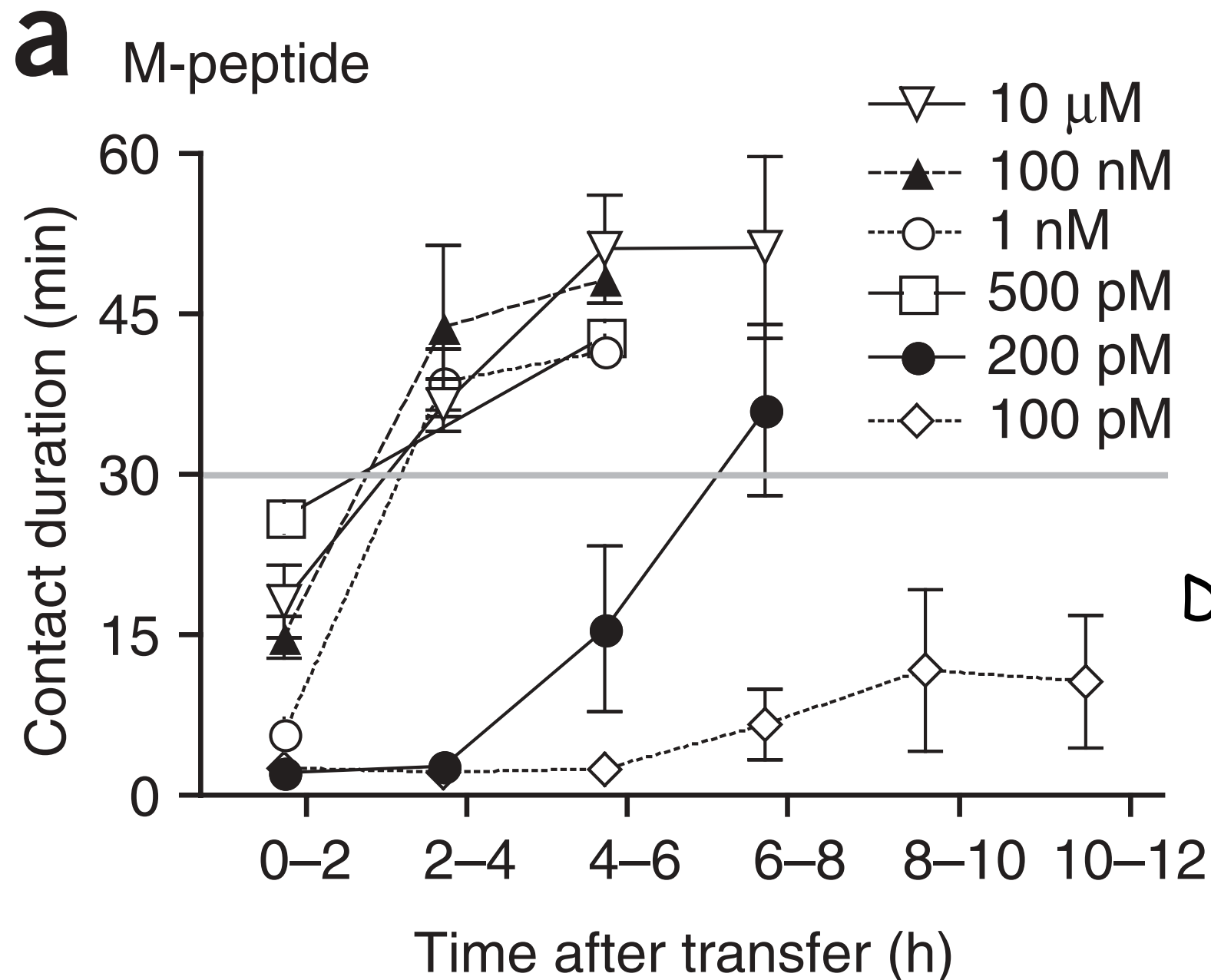
Careful: cells may die during later experiment steps

Quantification example 3 (most challenging): Contact times between specific T cells and DC



Green: Ag specific CD8 T cells, **Blue** control cells, and **Red** DC

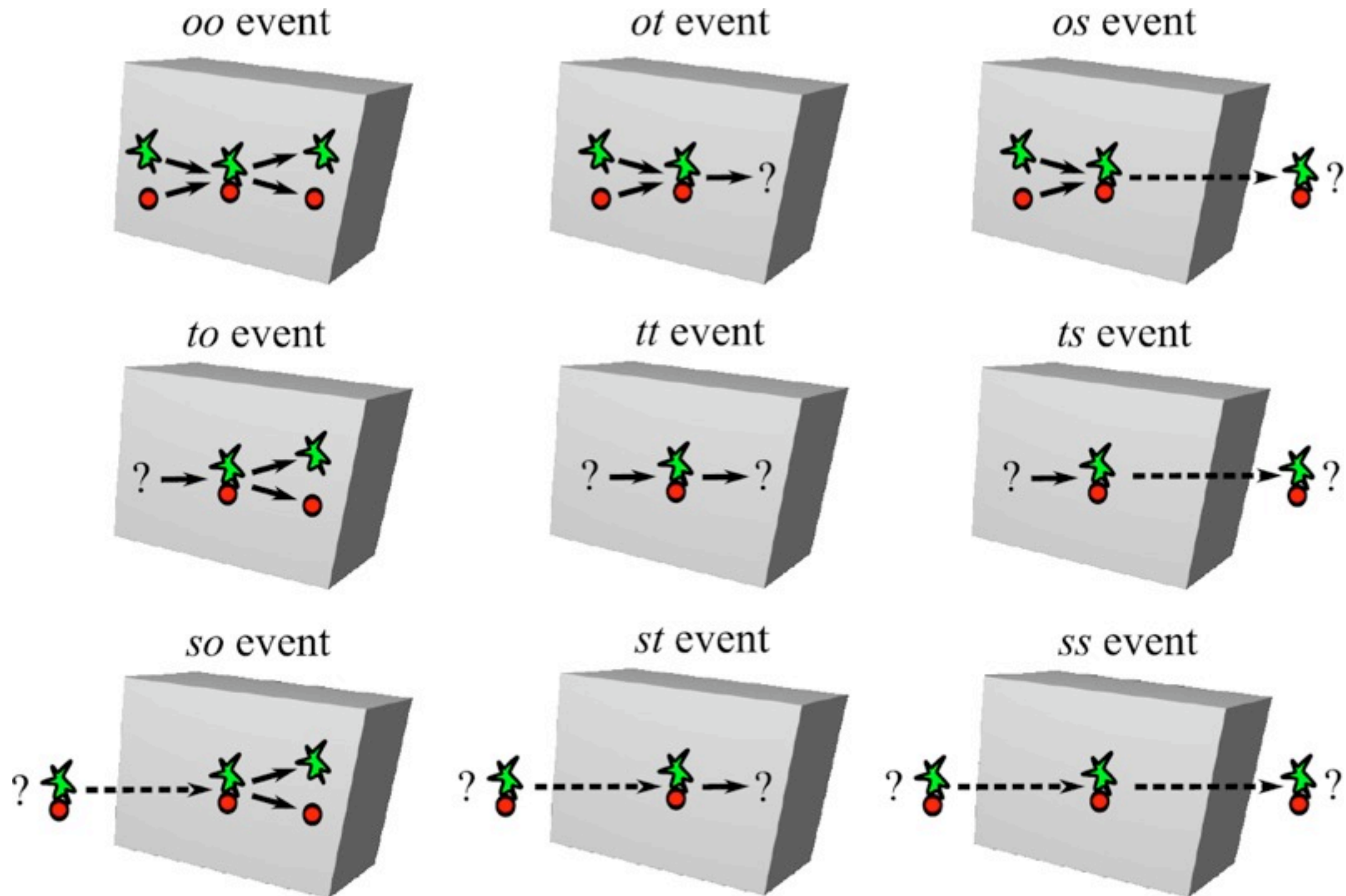
Observed contact times increase from phase 1 to 2



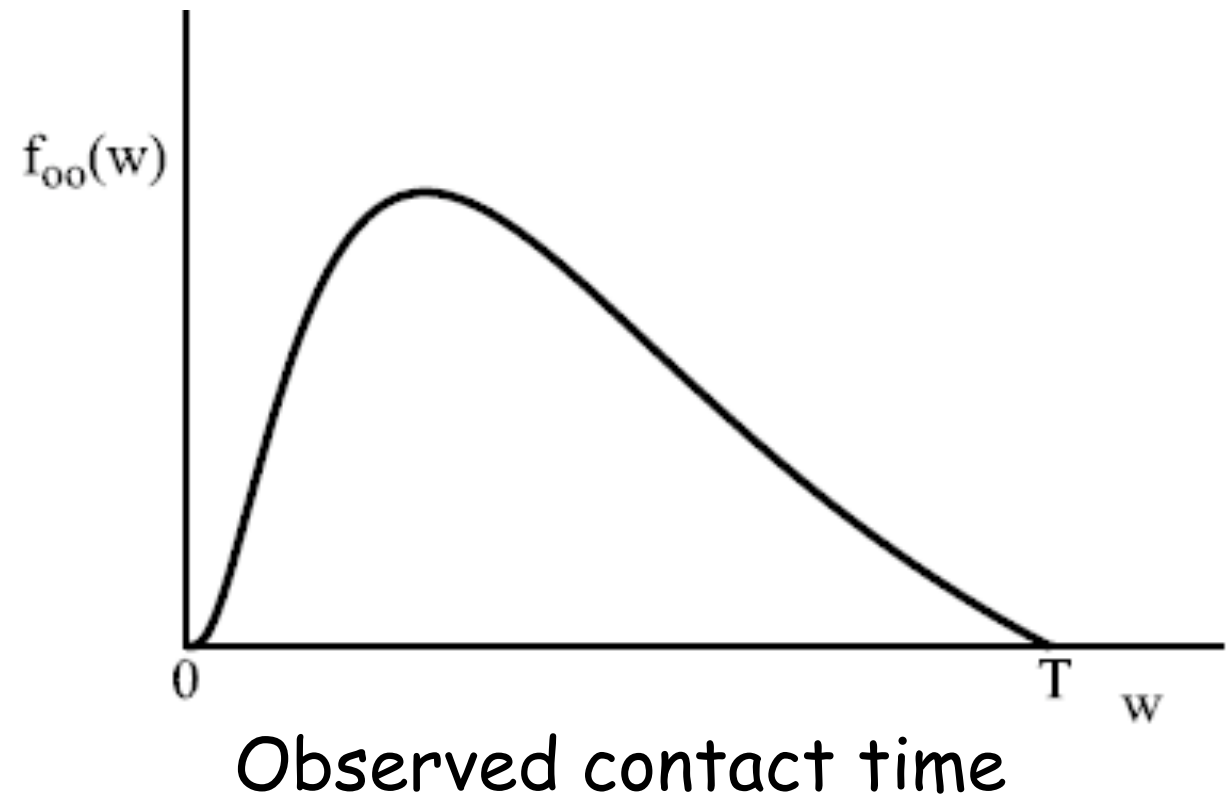
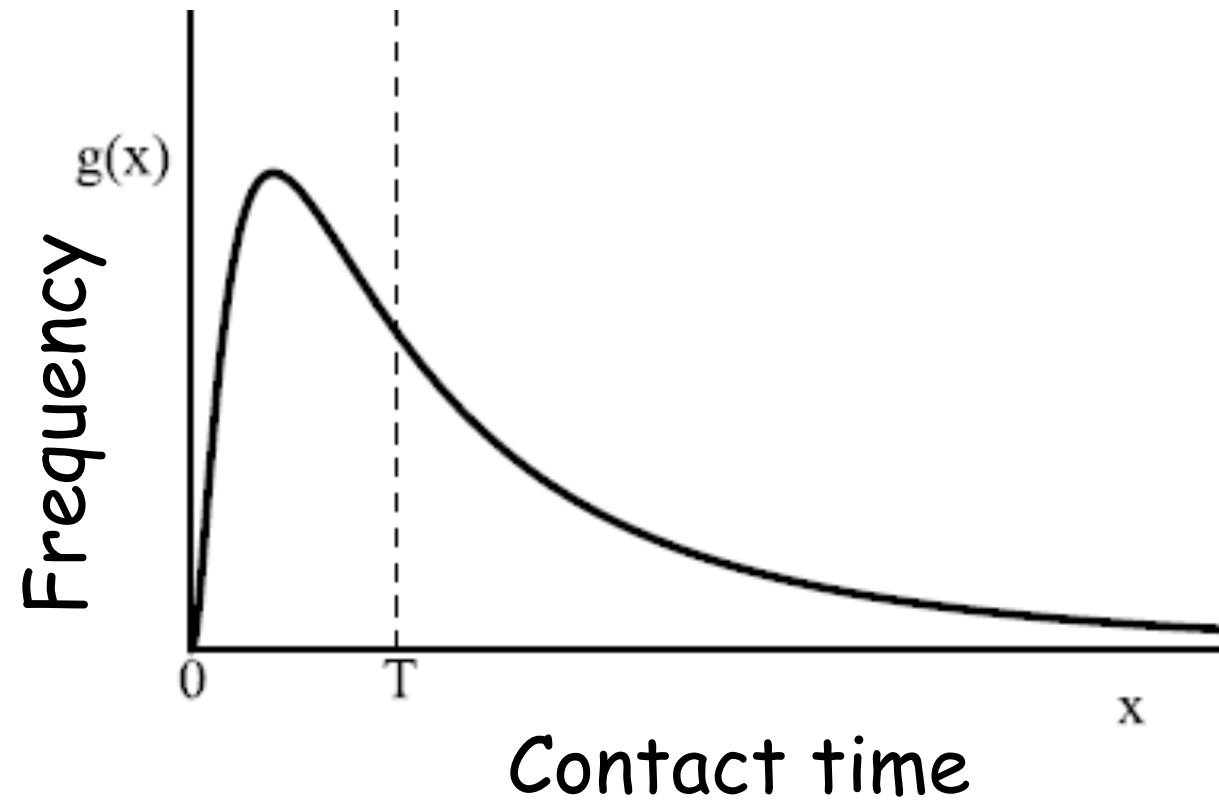
Data from Henrickson et al.
Nat Imm 2004

Movies typically last one hour which is shorter than many of the contacts in phase 2:
Difficult to estimate true contact times

From observed to true contact times: complicated problem



Assume a true contact distribution to predict the observed event distributions



True distribution $g(x)$ gives expected $f_{..}(w)$, where x is the true and w the observed contact time. T is imaging time and δ the rate of leaving the area δ is an average that is not expected to hold for cells that just entered the field

Compute probabilities to observe each event

J.B. Beltman et al. / Journal of Immunological Methods 347 (2009) 54–69

(a) *oo* event



to event



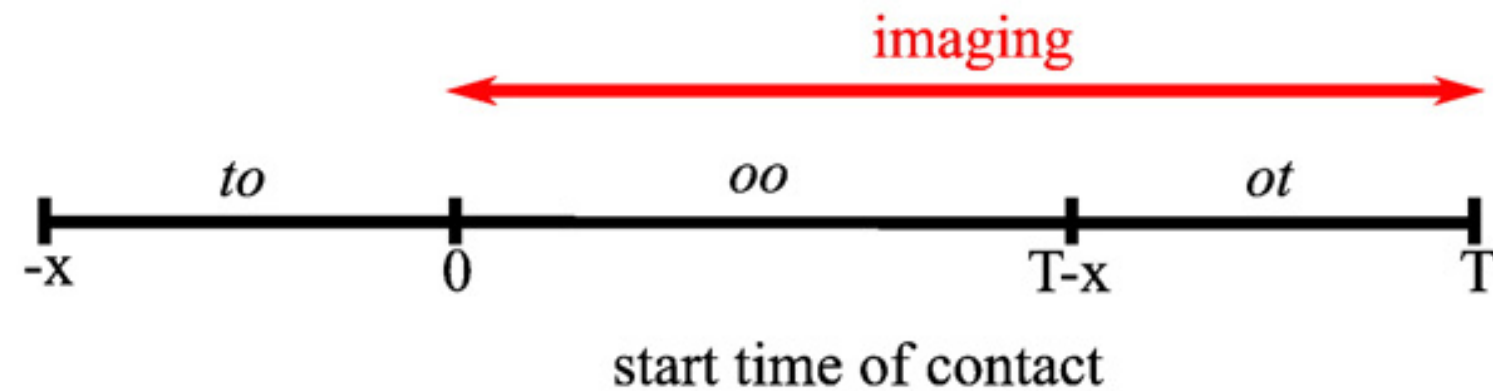
ot event



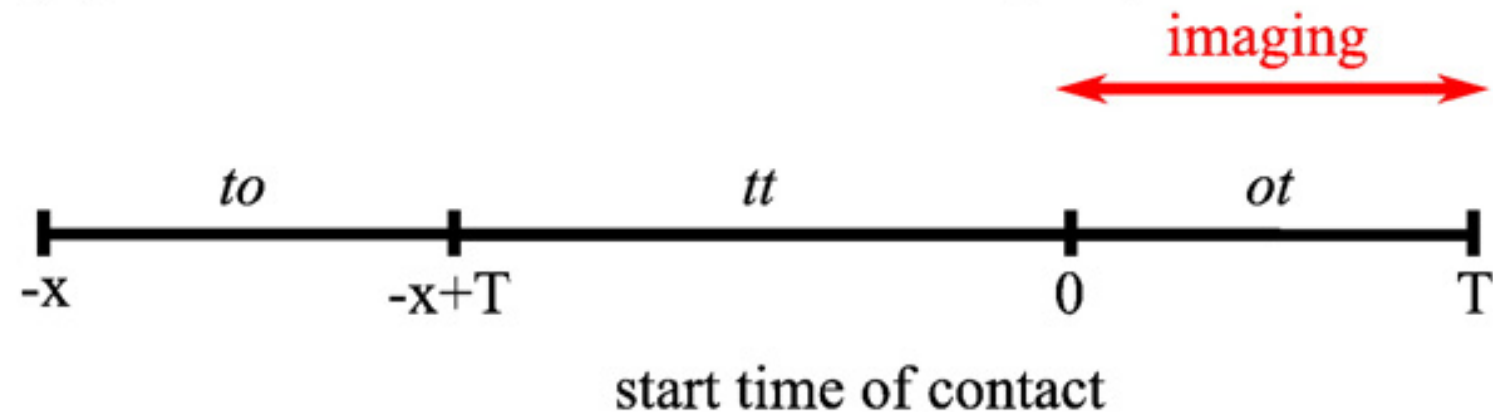
tt event



(b) true contact time < time window ($x < T$)



(c) true contact time > time window ($x > T$)



$$f_{oo}(w) = e^{-\delta w} (T - w) g(w)$$

$$f_{ot}(w) = f_{to}(w) = e^{-\delta w} \int_{x=w}^{\infty} g(x) dx ,$$

$$f_{tt}(T) = e^{-\delta T} \int_{x=T}^{\infty} (x - T) g(x) dx$$

Full model

$$f_{oo}(w) = e^{-\delta w}(T - w)g(w)$$

$$f_{ot}(w) = f_{to}(w) = e^{-\delta w} \int_{x=w}^{\infty} g(x)dx$$

$$f_{tt}(T) = e^{-\delta T} \int_{x=T}^{\infty} (x - T)g(x)dx$$

$$f_{os}(w) = f_{so}(w) = \delta e^{-\delta w}(T - w) \int_{x=w}^{\infty} g(x)dx$$

$$f_{ts}(w) = f_{st}(w) = \delta e^{-\delta w} \int_{x=w}^{\infty} (x - w)g(x)dx$$

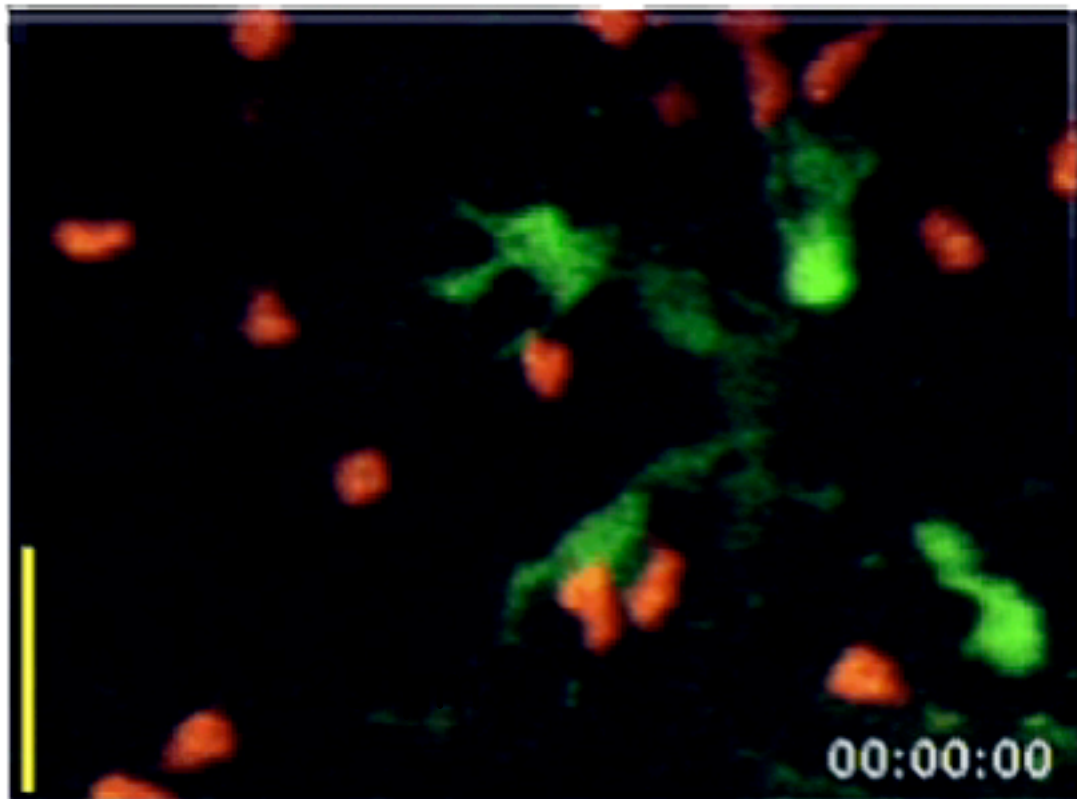
$$f_{ss}(w) = \delta^2 e^{-\delta w}(T - w) \int_{x=w}^{\infty} (x - w)g(x)dx$$

$$f_{oo}(w) + \dots + f_{ss}(w) = \int_{x=0}^{\infty} (T + (1 + \delta T)x)g(x)dx$$

The latter gives the total number of events one expects to observe, normalized to the total number of contacts, N , initiated per hour.

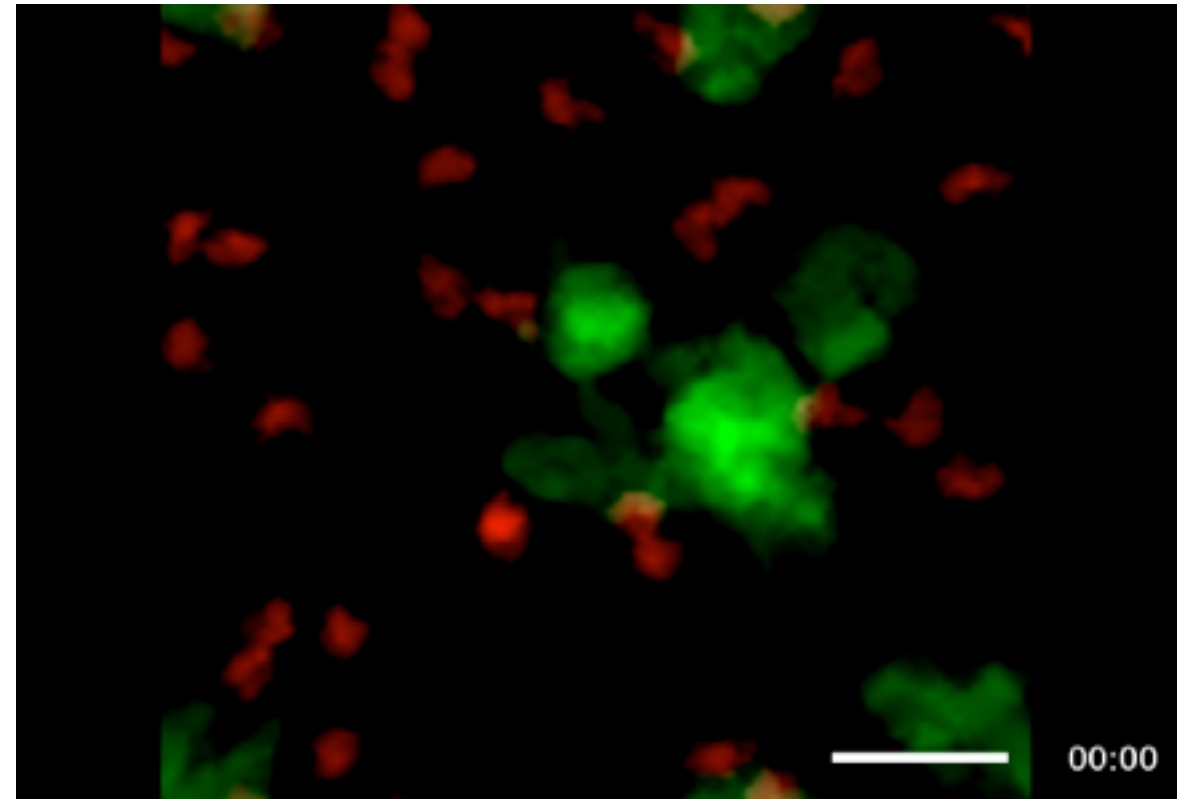
First test the method with our CPM

in vivo



Miller et al. J Exp Med (2004)

in silico (CPM)

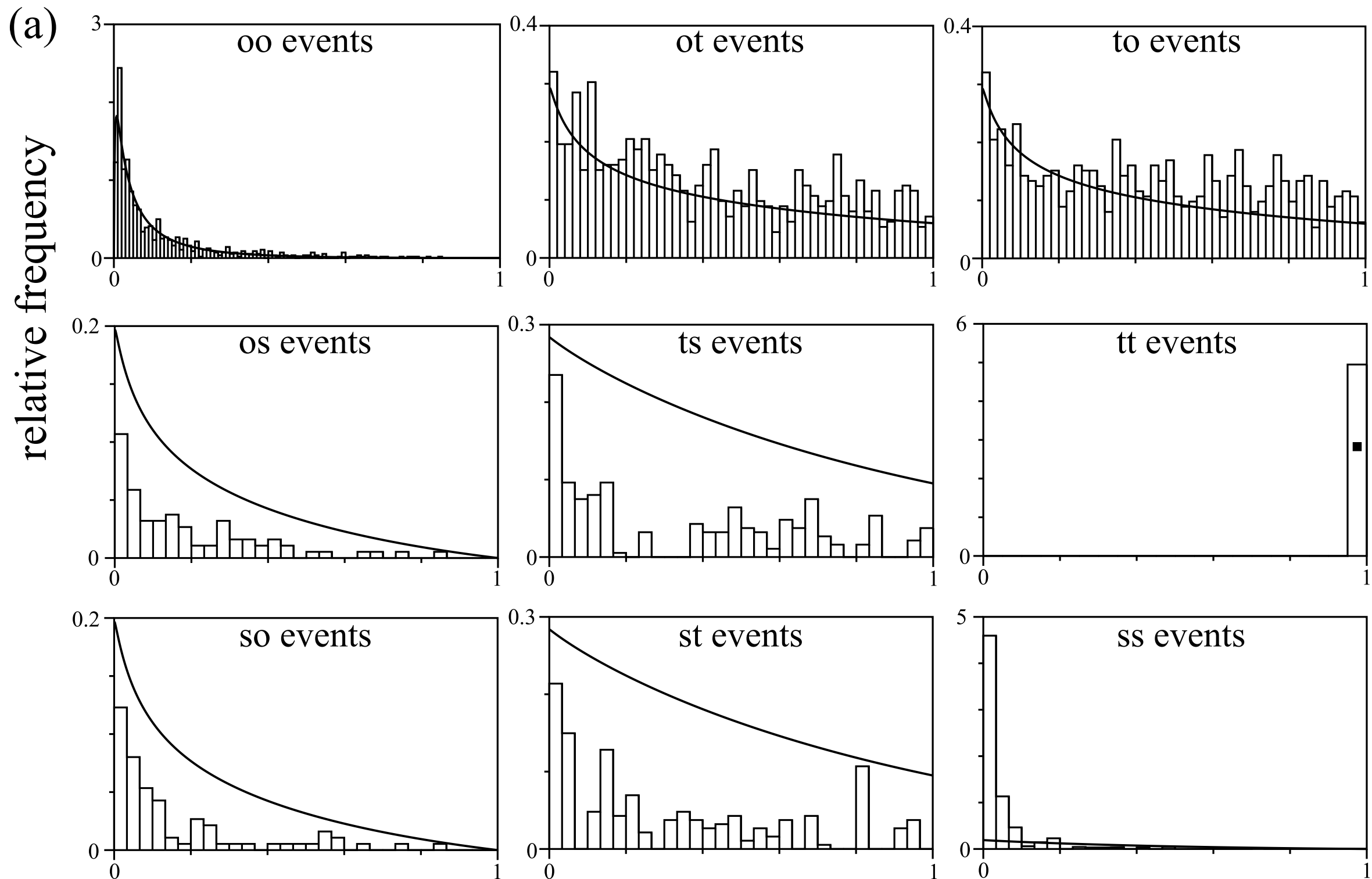


Beltman et al. J Exp Med (2007)

Red: T cells **Green:** Dendritic cells (DC)

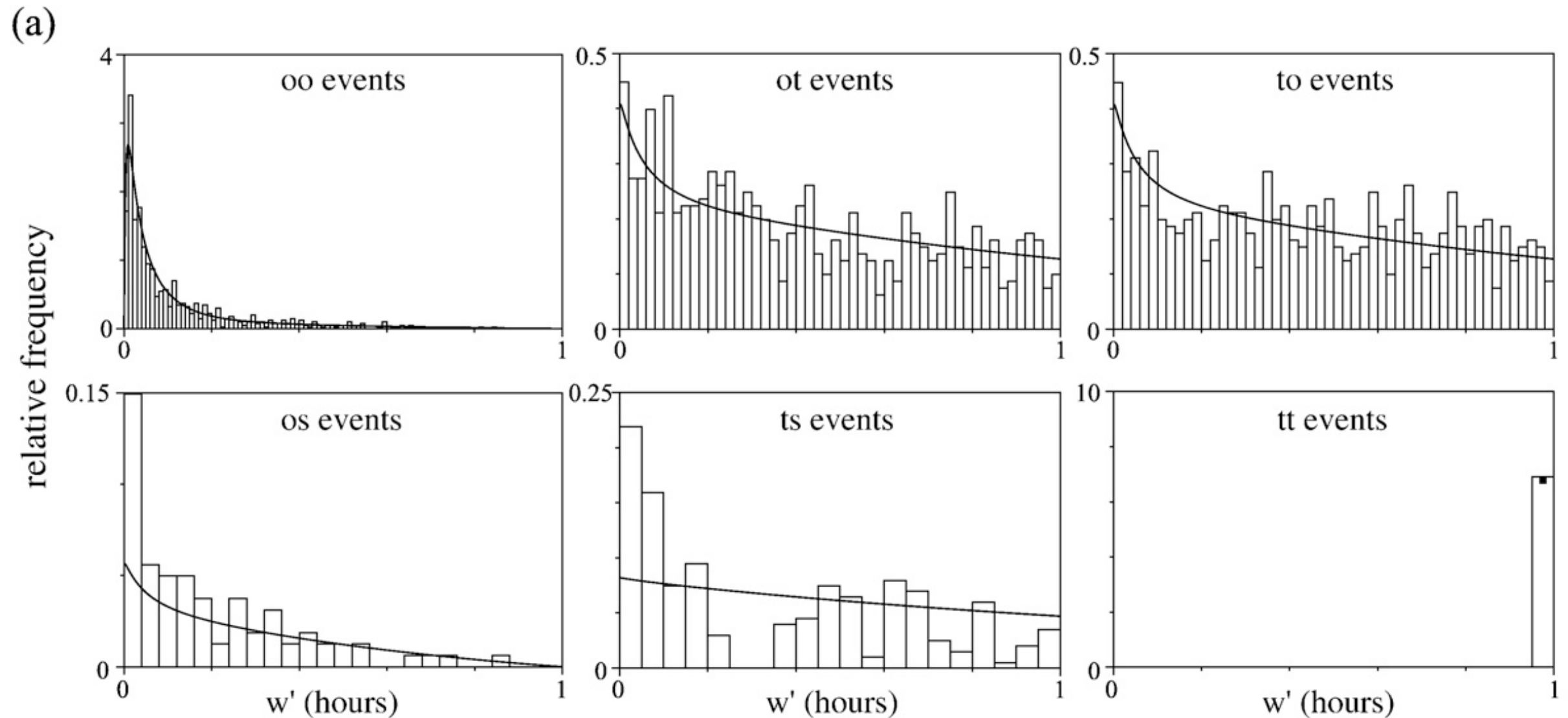
Computer model (CPM) with realistic behavior

Validate method using CPM simulations



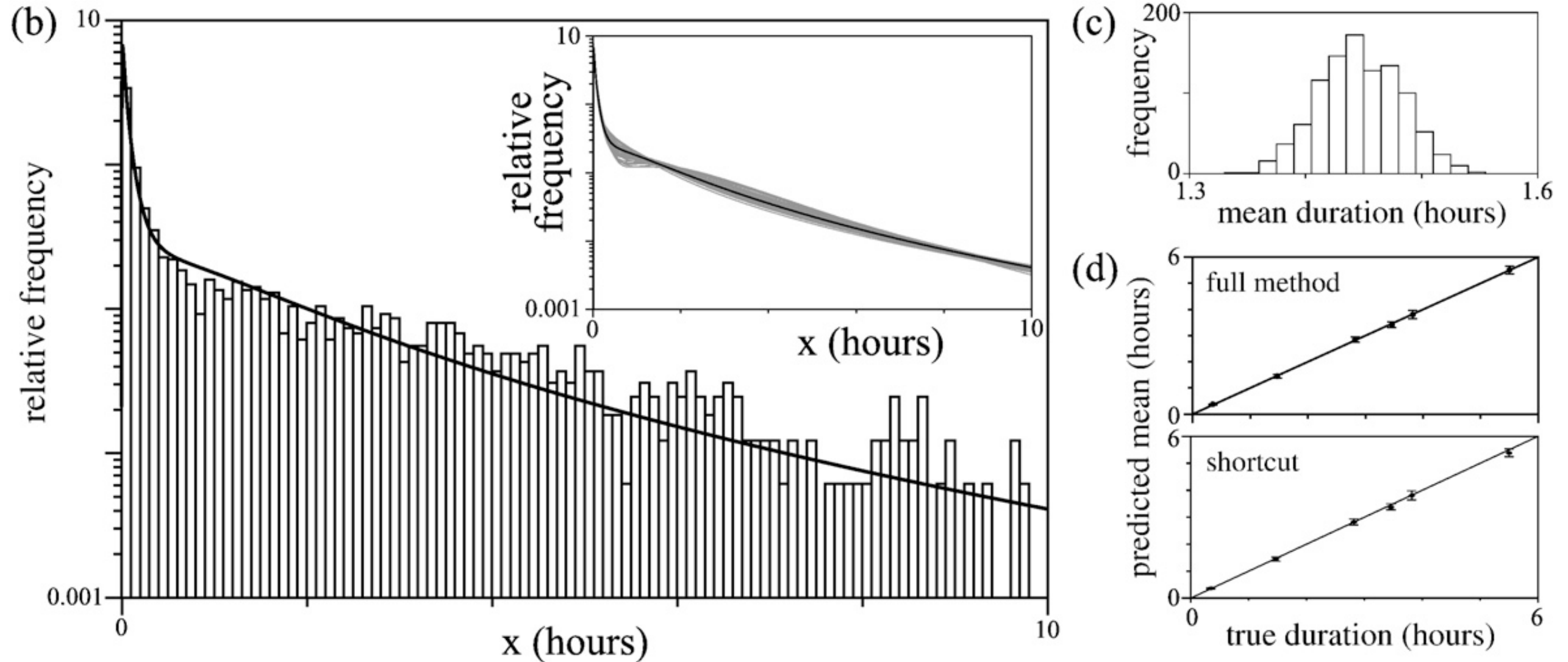
Split 100-h simulations into 100 parts of 1 hour
fit observed events by maximum likelihood procedure

Ignoring all events of entering cells (t.)



gives a much better description of the in silico data

and a correct estimate of the contact times



fitting the sum of two lognormals for $g(x)$

Shortcut method

Total number of conjugates at any point in time:

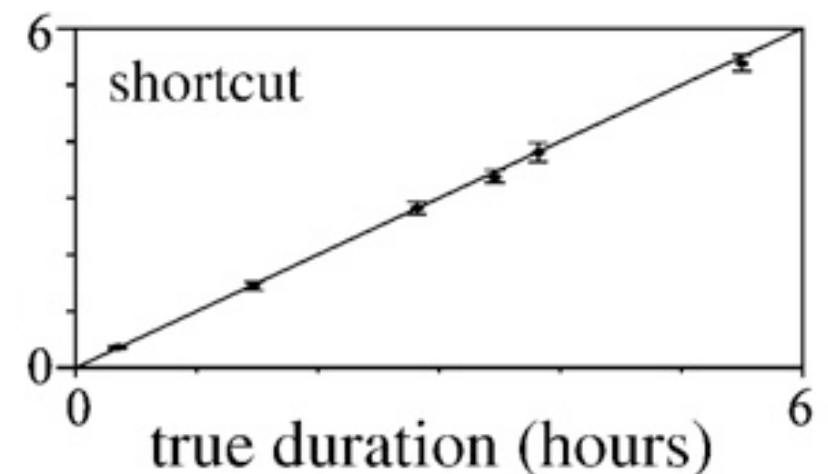
$$\overline{n_C} = N \int_{x=0}^{\infty} x g(x) dx$$

where N is the number of contacts initiated per hour.

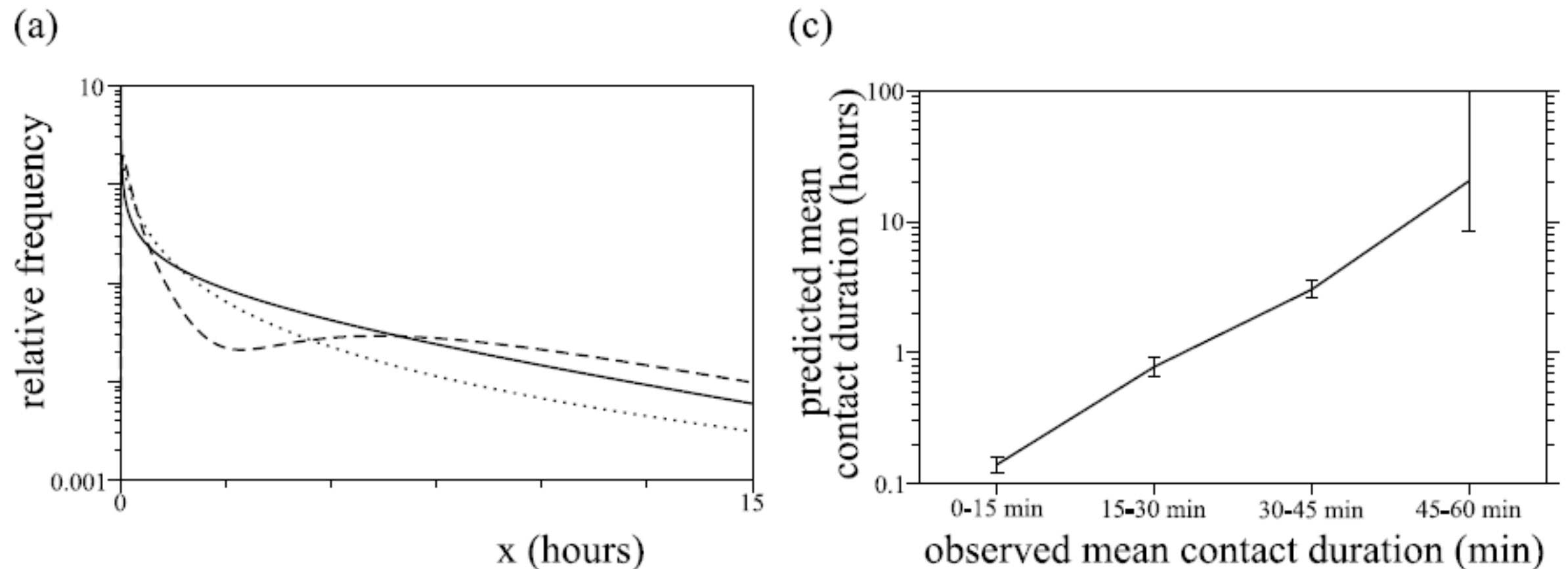
The average contact time can thus be calculated by dividing the (average) number of conjugates by the number of initiation events:

$$\bar{x} = \frac{2T\overline{n_C}}{n_i + n_t}$$

(excluding entries and exits)



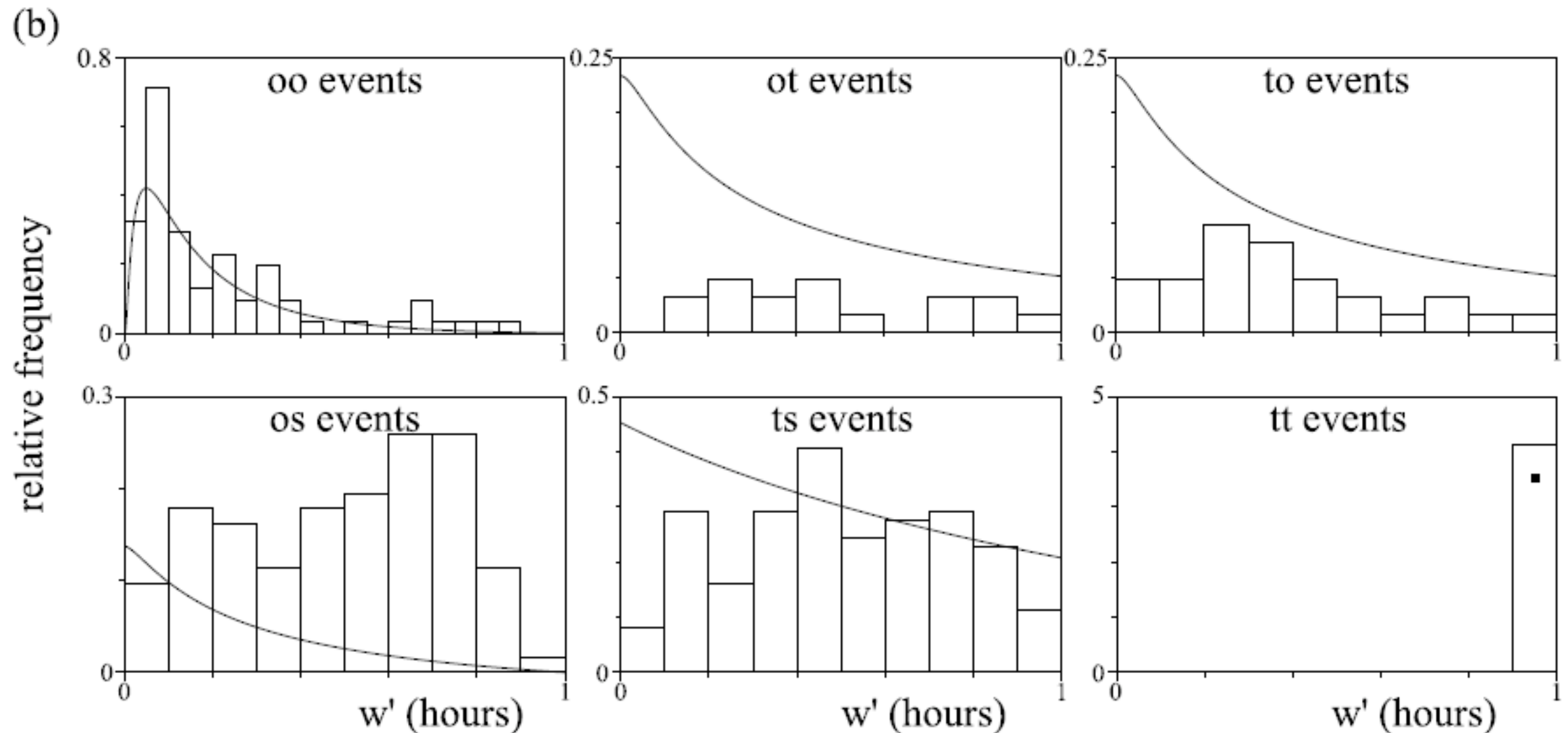
Contact data from Henrickson Nat Imm 2008



Assuming either a gamma distribution for $g(x)$ (solid line), a lognormal (dotted), or the sum of two lognormals (dashed), we estimate very similar average contact times

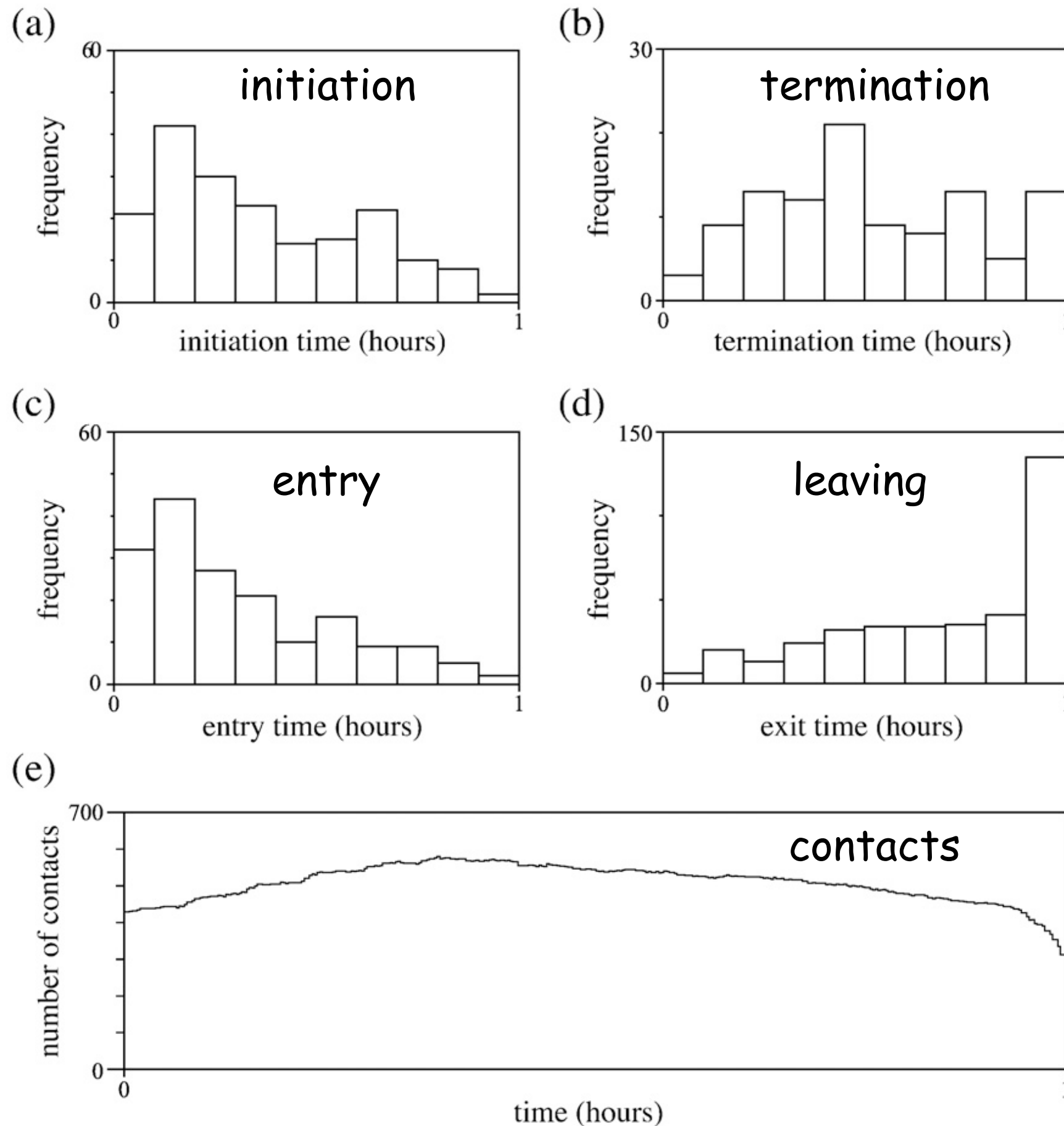
**Fits suggest an exponential relation
between observed and true contact time**

But fits are of poor quality



to, ot, os and ts events have to decline with the observed contact duration, but do not.

Check data for artifacts: tissue drift



Observed initiation (a), termination (b), entry (c) and leaving (d) events, and number of contacts should be constant over time.

Subset of 9/33 expts suggest $x=5h$

Artefact or bias		How to detect	How to correct	
S S		T (C	M C	
D		C d	M n	
E n		P f a	D	
I C	z	P a (f t s	M t d	z
S		I i o a	S p m	
C p b		N	U s c b m s	
T t C O		N	E f C S	4. 4.

Conclusions

Parameter estimation is far from trivial

Observed contact times are biased by restricted time window and spatial area

True contact times can be estimated by fitting a complicated maximum likelihood model and/or by a simple shortcut method

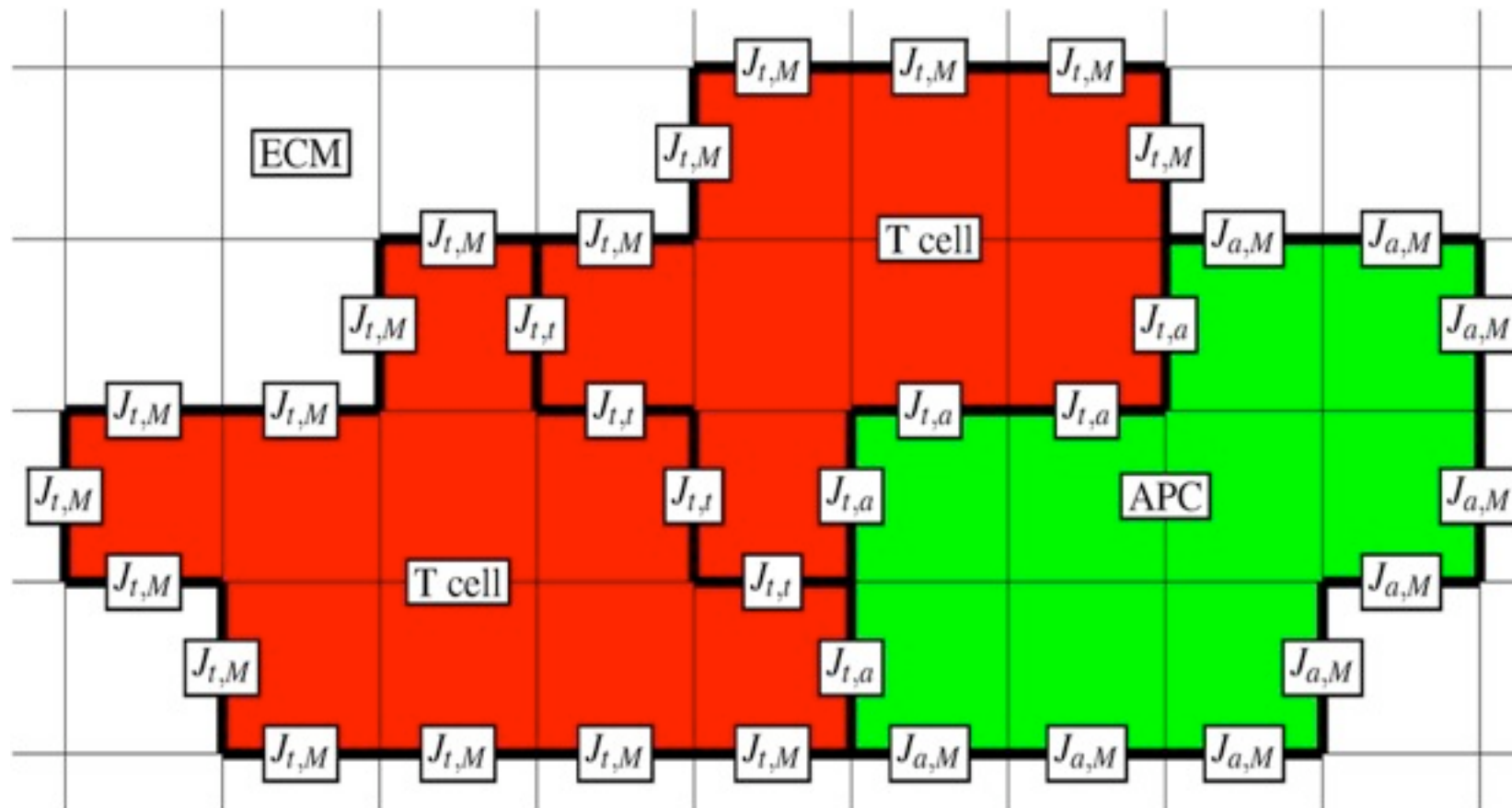
All data: 3.3h (2.8-3.8), Best data: 5h (1.7-7)

Beltman et al. J Immunol Methods 2009

Test data for artifacts like tissue drift

Beltman et al. Nature Revs Immunol 2009

Cellular Potts Model: grid



Cells have a target volume

Matrix of adhesion coefficients J between all cell types
asynchronous Cellular Automaton

Surface energies: Hamiltonian

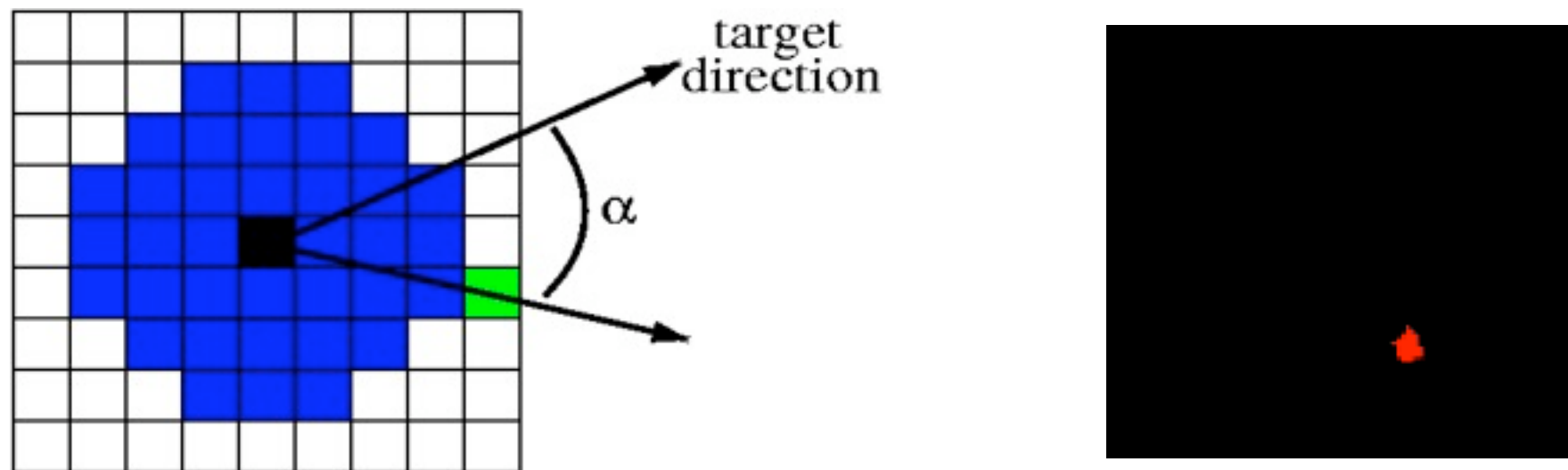
System minimizes its energy

ΔH determines probability of copying

$$H = \sum J + \lambda (v - V_T)^2$$

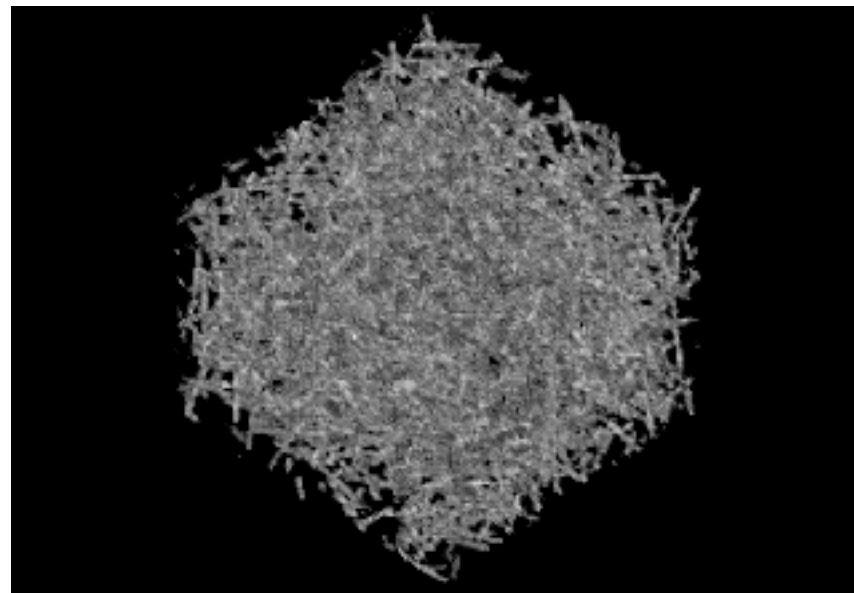
T cells: target direction

$$\Delta H = -\mu \cos(\alpha)$$



Adjust target direction according to recent displacement
(also directional persistence)

Model T cell area in LN: RT network



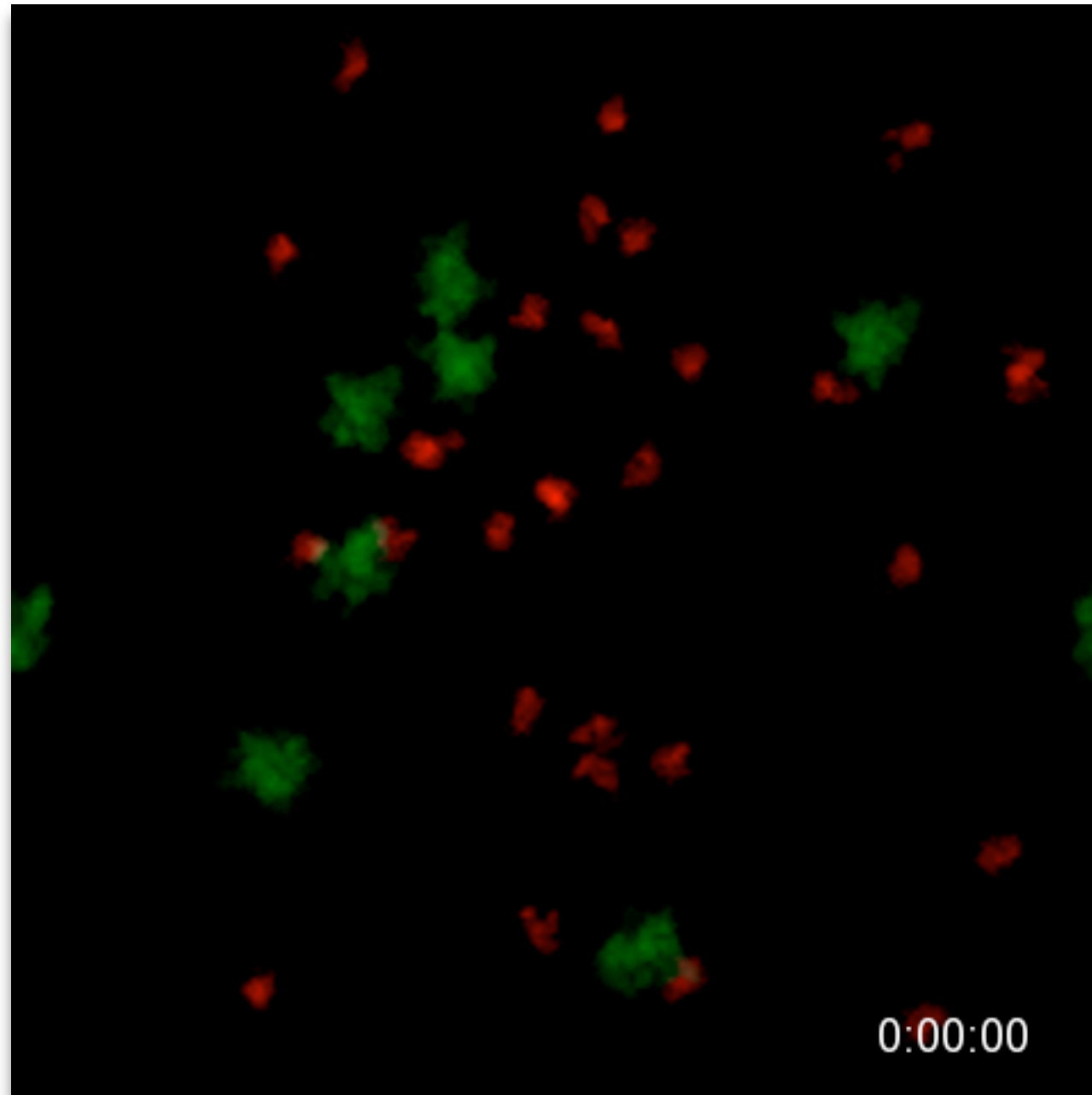
1 pixel = $1 \mu\text{m}^3$

T cell: $150 \mu\text{m}^3$, DC: $2200 \mu\text{m}^3$

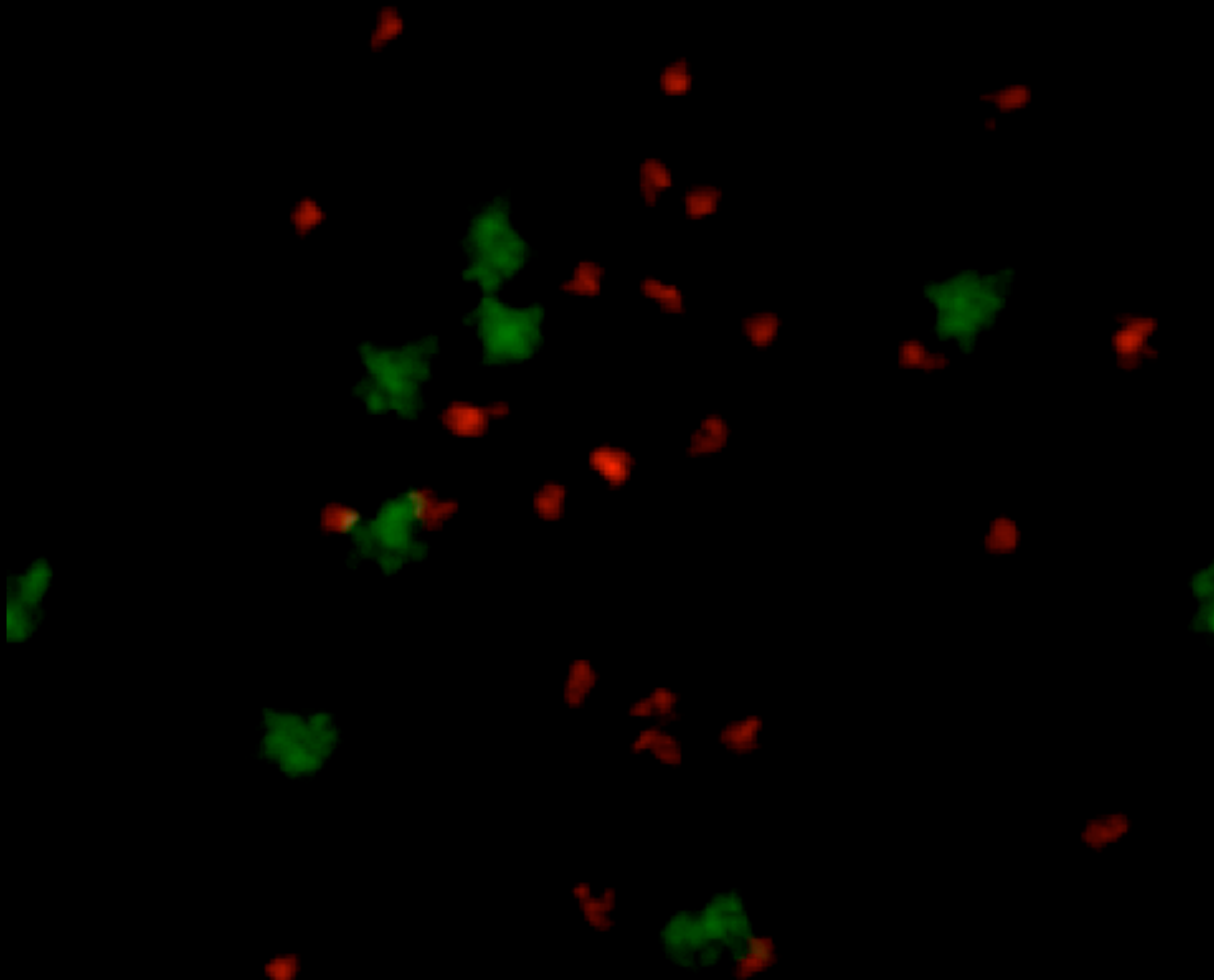
torus: $100 \mu\text{m} \times 100 \mu\text{m} \times 100 \mu\text{m}$

static reticular network (rods)

Now with Antigen: red Ag specific
T cells, green cognate DCs



During short contacts cells increase their adhesion for
APCs, between contacts they slowly forget this



0:00:00